

A theoretical scheme for generation of Gazeau-Klauder coherent states via intensity-dependent degenerate Raman interaction

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Abstract

A theoretical scheme is presented for generating Gazeau-Klauder coherent states (GKCSs) via the generalization of degenerate Raman interaction with coupling constant to intensity-dependent coupling. Firstly, we prove that in the intensity-dependent degenerate Raman interaction, under particular conditions, the modified effective Hamiltonian can be used instead of Hamiltonian in the interaction picture, for describing the atom-field interaction. We suppose that the cavity field is initially prepared in a nonlinear CS, which is not temporally stable. As we will observe, after the occurrence of the interaction between atom and field, the generated state involves a superposition of GKCSs which are temporally stable and initial nonlinear CS. Under specific conditions which may be prepared, the generated state just includes GKCS. So, in this way we produced the GKCS, successfully.

Keywords: Generation of Gazeau-Klauder coherent state, Nonlinear coherent states, Degenerate Raman interaction, Modified effective Hamiltonian.

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1 Introduction

Coherent states (CSs) defined as the right eigenstates of the harmonic oscillator annihilation operator, i.e., $a|\alpha\rangle = \alpha|\alpha\rangle$ play an important role in quantum optics and modern physics [1]. Along the generalization of these states, nonlinear CSs [2] or f -CSs [3] have been introduced. According to this formalism f -deformed annihilation and creation operators, respectively defined as $A = af(n)$ and $A^\dagger = f^\dagger(n)a^\dagger$ where a, a^\dagger and $n = a^\dagger a$

are bosonic annihilation, creation and number operators, respectively. The intensity-dependent function $f(n)$ is responsible for the nonlinearity of the states. Nonlinear CSs $|z, f\rangle$ are then defined as the right eigenstates of f -deformed annihilation operator,

$$A|z, f\rangle = z|z, f\rangle. \quad (1)$$

The Fock space representation of these states is explicitly given by

$$\begin{aligned} |z, f\rangle &= \sum_{n=0}^{\infty} C_n |n\rangle, \quad C_n = \mathcal{N}(|z|^2) \frac{z^n}{\sqrt{n!} [f(n)]!}, \\ \mathcal{N}(|z|^2) &= \left(\sum_{n=0}^{\infty} \frac{|z|^{2n}}{n! ([f(n)]!)^2} \right)^{-1/2}, \quad z \in \mathbb{C} \end{aligned} \quad (2)$$

where $[f(n)]! = f(1)f(2)\dots f(n)$ and by convention $[f(0)]! \doteq 1$. Note that we have confined ourselves to the special case of real valued function $f(n)$.

Roknizadeh et al derived a Hamiltonian associated to a nonlinear system based on action identity requirement of nonlinear coherent states as $H = A^\dagger A = n f^2(n)$ [4]. So, the eigenvalue equation for any one dimensional physical system with known discrete eigenvalues may be given by $H|n\rangle = e_n|n\rangle = n f^2(n)|n\rangle$, where $0 = e_0 < e_1 < e_2 < \dots < e_n < e_{n+1} < \dots$. Accordingly, one simply has $f(n) = \sqrt{e_n/n}$ associated to solvable quantum systems. In this way the nonlinear CSs may be deduced corresponding to any solvable quantum system as [5]:

$$|z, e_n\rangle = \mathcal{N}(|z|^2) \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{[e_n]!}} |n\rangle. \quad (3)$$

But it is remarkable that these states are not temporally stable like the original nonlinear CSs in (2). Moreover, an analytical representation of Gazeau-Klauder CSs (GKCSs) corresponding to any Hamiltonian with discrete (nondegenerate) eigenvalues is defined as [6, 7, 8]

$$|z, \alpha\rangle = \mathcal{N}(|z|^2) \sum_{n=0}^{\infty} \frac{z^n e^{-i\alpha e_n}}{\sqrt{\rho(n)}} |n\rangle, \quad z \in \mathbb{C}, \quad \alpha \in \mathbb{R}, \quad (4)$$

where $\mathcal{N}(|z|^2)$ is a normalization constant. These states satisfy the following requirements: (i) continuity of label, (ii) resolution of the identity, (iii) temporal stability and (iv) action identity. The last condition requires $\rho(n) = [e_n]!$. So the state in (4) can be written as

$$|z, \alpha\rangle = \mathcal{N}(|z|^2) \sum_{n=0}^{\infty} \frac{z^n e^{-i\alpha e_n}}{\sqrt{[e_n]!}} |n\rangle. \quad (5)$$

It is also established in [7] that GKCSs are a class of nonlinear CSs with nonlinearity function $f_{GK}(\alpha, n) = \sqrt{\frac{e_n}{n}} e^{i\alpha(e_n - e_{n-1})}$, which are temporally stable, i.e.,

$$e^{-iHt}|z, \alpha\rangle = |z, \alpha'\rangle, \quad \alpha' \equiv \alpha + t. \quad (6)$$

Along this work, Gazeau-Klauder squeezed states associated to solvable quantum systems have been introduced by one of us [9].

On the other hand, recently there has been much interest in the superpositions of CSs [10]. Due to the quantum interference between the coherent components, such superposition states may exhibit various nonclassical properties such as squeezing and sub-Poissonian statistics. Nonclassical states of light are central to quantum optics. Their importance comes from potential applications on advanced optics, as teleportation [11], quantum computation [12], quantum communication [13], quantum cryptography [14], quantum lithography [15], etc. A number of schemes have been proposed for generating such states [16]. Among them, a method has been presented for generating superpositions of CSs of a cavity field via degenerate Raman interaction [17, 18]. In this paper, we have followed the same approach and suggested a theoretical scheme for generating the GKCSs which are temporally stable, via the intensity-dependent degenerate Raman interaction. It is worth to mention that, we have used the intensity-dependent modified effective Hamiltonian which describes appropriately the dynamics of the interaction between atom and field. This is while, to the best of our knowledge, no scheme for generation of GKCSs may be found in the earlier literature.

2 Degenerate Raman interaction and modified effective Hamiltonian

The degenerate Raman interaction describes the interaction between a degenerate Λ -type three-level atom (as shown in Fig. 1) and a single-mode radiation field characterized by bosonic operators a, a^\dagger . Various properties of this system have been previously discussed [19, 20]. The Hamiltonian for such a system in the interaction picture is given by:

$$H_I = g_1(a^\dagger|g\rangle\langle i|e^{-i\Delta t} + a|i\rangle\langle g|e^{i\Delta t}) + g_2(a^\dagger|e\rangle\langle i|e^{-i\Delta t} + a|i\rangle\langle e|e^{i\Delta t}), \quad (7)$$

where we have set $\hbar \equiv 1$, $|e\rangle$ and $|g\rangle$ are the two degenerate lower states and $|i\rangle$ is the upper state of the atom, the coupling constant of the transition between $|i\rangle$ and $|g\rangle(|e\rangle)$ with the cavity field is denoted by $g_1(g_2)$, $\Delta = (\omega_i - \omega_0) - \omega_f$ is detuning, where ω_f is the cavity field frequency, ω_0 is the energy of the two lower states and ω_i is the energy for the upper level of the atom. On the other side, to describe the degenerate Raman interaction, an effective Hamiltonian was derived by Agarwal in [21], when the atomic transition between the upper and lower levels is far from the frequency of the field mode. Indeed, after adiabatically eliminating the upper level, the author introduced the effective Hamiltonian as:

$$H_e = -\lambda a^\dagger a (|g\rangle\langle e| + |e\rangle\langle g|), \quad (8)$$

where $\lambda = g_1 g_2 / \Delta$ indicates the effective atom-field coupling constant. In fact, in the effective Hamiltonian approach, the two lower levels are coupled with a single-mode field

through a virtual upper level. As a result, the degenerate Raman interaction is reduced to an effective two-level system. Also, the effective Hamiltonian given in (8) leads to the elimination of time-dependent phase factors in (7). These factors are related to the ac Stark shifts of energy levels [22]. The Hamiltonian that represents the Stark shifts of energy levels is given by:

$$H_s = -a^\dagger a (\lambda_1 |g\rangle\langle g| + \lambda_2 |e\rangle\langle e|), \quad (9)$$

where $\lambda_1 = g_1^2/\Delta$ ($\lambda_2 = g_2^2/\Delta$) describes the Stark shift of the state $|g\rangle$ ($|e\rangle$). Hence, one may modify the effective Hamiltonian given in (8), so that it includes the ac Stark effects represented in (9). In this way, for the modified effective Hamiltonian, one has then [23]

$$H_{eff} = H_e + H_s. \quad (10)$$

Inserting H_e and H_s respectively from (8) and (9) in (10), the modified effective Hamiltonian can be written as follows:

$$H_{eff} = -\lambda a^\dagger a (|g\rangle\langle e| + |e\rangle\langle g|) - a^\dagger a (\lambda_1 |g\rangle\langle g| + \lambda_2 |e\rangle\langle e|). \quad (11)$$

It is found that, the modified effective Hamiltonian H_{eff} is more suitable than the well-known effective Hamiltonian H_e in studying the time evolution of the systems. Xu and Zhang in [23] showed that, in the degenerate Raman interaction, far off-resonant (enough large value of Δ), the modified effective Hamiltonian has been given in (11), can be applied instead of the Hamiltonian in the interaction picture in (7). In fact, starting from a unique state vector at $t = 0$, both of the Hamiltonians lead to the same state vector of atom-field system at any time t . Altogether, several other mathematical physics methods were suggested to obtain the modified effective Hamiltonian in [24, 25].

3 Generalizing the degenerate Raman interaction to intensity-dependent interactions

Generalization of the single-mode to two-mode degenerate Raman process has been recently done in [26]. But, to achieve our aim of the paper, we should generalize the single-mode degenerate Raman interaction in a rather different way. Indeed, following the path of Knight in [27] and specifically Zheng in [28], we replace the bosonic operators a, a^\dagger in (11) with $A = a f(n), A^\dagger = f(n)a^\dagger$, i.e., the generalized f -deformed ladder operator. This type of generalization may be arisen from the same procedure of degenerate Raman interaction, briefly illustrated in the previous section, with the difference that the interaction between the same Λ -type atom and a single-mode radiation field is now

considered to be intensity-dependent. Henceforth, analogously to (7) one may suggest the following form for the Hamiltonian of such a system in the interaction picture

$$\mathcal{H}_I = g_1(A^\dagger|g\rangle\langle i|e^{-i\Delta t} + A|i\rangle\langle g|e^{i\Delta t}) + g_2(A^\dagger|e\rangle\langle i|e^{-i\Delta t} + A|i\rangle\langle e|e^{i\Delta t}). \quad (12)$$

In the same way, analogously to (11), for the intensity-dependent modified effective Hamiltonian we suggest

$$\mathcal{H}_{eff} = -\lambda A^\dagger A (|g\rangle\langle e| + |e\rangle\langle g|) - A^\dagger A (\lambda_1|g\rangle\langle g| + \lambda_2|e\rangle\langle e|). \quad (13)$$

Clearly, inserting $f(n) = 1$ in (12) and (13) recovers the usual degenerate Raman interaction (intensity independent coupling) introduced in (7) and (11), respectively. This type of generalization (of degenerate Raman interaction) to intensity-dependent interaction can be frequently found in the literature, for instance, whenever one imposes nonlinearity on the standard JCM [29].

Before we proceed, to establish the above formal approach more explicitly, the following discussion may be offered. As mentioned, when Δ is enough large, the equivalence of the introduced Hamiltonian in (7) and (11) are demonstrated in [23]. Our aim at this stage is to prove that, in the nonlinear degenerate Raman interaction, under particular conditions, the nonlinear modified effective Hamiltonian in (13) can be applied appropriately instead of the nonlinear Hamiltonian in the interaction picture introduced in (12). To investigate the validity of our proposal, we will obtain the time evolution of the state vector of the atom-field system by using (12) and (13), and continue with showing that for enough large value of Δ , these two state vectors are equal. For this purpose, we suppose that the atom is initially in a superposition of $|g\rangle$ and $|e\rangle$:

$$|\Phi_a(t=0)\rangle = C_g(0)|g\rangle + C_e(0)|e\rangle, \quad (14)$$

and the field is in the arbitrary state:

$$|\Phi_F(t=0)\rangle = \sum_{n=0}^{\infty} q_n |n\rangle, \quad (15)$$

where q_n determines the initial state of the field. The state of the atom-field system at $t = 0$ is thus given by:

$$|\Phi_s(t=0)\rangle = \sum_{n=0}^{\infty} q_n (C_g(0)|g, n\rangle + C_e(0)|e, n\rangle). \quad (16)$$

The time evolution of the state of the atom-field system, governed by \mathcal{H}_I introduced in (12), is the solution of:

$$i\frac{\partial}{\partial t}|\Phi_{s,I}(t)\rangle = \mathcal{H}_I|\Phi_{s,I}(t)\rangle, \quad (17)$$

where $|\Phi_{s,I}(t)\rangle$, denotes the state of the atom-field system after time t . The following general solution may be considered as:

$$|\Phi_{s,I}(t)\rangle = \sum_{n=0}^{\infty} q_n [C_g^n(t)|g, n\rangle + C_i^n(t)|i, n-1\rangle + C_e^n(t)|e, n\rangle]. \quad (18)$$

The problem is now determining the time dependent coefficients which may be obtained by lengthy but straightforward calculations with the following final results:

$$C_g^n(t) = A_1^n(t) C_g(0) + A_2^n(t) C_e(0), \quad (19)$$

$$C_i^n(t) = B_1^n(t) C_g(0) + B_2^n(t) C_e(0), \quad (20)$$

$$C_e^n(t) = A_2^n(t) C_g(0) + A_3^n(t) C_e(0), \quad (21)$$

where we have set

$$A_1^n(t) = e^{-i\Delta t/2} \left(\frac{g_2^2}{G} e^{i\Delta t/2} + \frac{g_1^2}{G} \cos \Lambda_n t + i \frac{\Delta g_1^2}{2\Lambda_n G} \sin \Lambda_n t \right), \quad (22)$$

$$A_2^n(t) = e^{-i\Delta t/2} \left(-\frac{g_1 g_2}{G} e^{i\Delta t/2} + \frac{g_1 g_2}{G} \cos \Lambda_n t + i \frac{\Delta g_1 g_2}{2\Lambda_n G} \sin \Lambda_n t \right), \quad (23)$$

$$A_3^n(t) = e^{-i\Delta t/2} \left(\frac{g_1^2}{G} e^{i\Delta t/2} + \frac{g_2^2}{G} \cos \Lambda_n t + i \frac{\Delta g_2^2}{2\Lambda_n G} \sin \Lambda_n t \right), \quad (24)$$

$$B_1^n(t) = e^{i\Delta t/2} \left(-i \frac{g_1 \sqrt{n} f(n)}{\Lambda_n} \sin \Lambda_n t \right), \quad (25)$$

$$B_2^n(t) = e^{i\Delta t/2} \left(-i \frac{g_2 \sqrt{n} f(n)}{\Lambda_n} \sin \Lambda_n t \right), \quad (26)$$

with the following definitions:

$$\Lambda_n \doteq \sqrt{n f^2(n) G + \frac{\Delta^2}{4}}, \quad (27)$$

$$G \doteq g_1^2 + g_2^2. \quad (28)$$

On the other side, if we use \mathcal{H}_{eff} introduced in (13) for obtaining the time evolution of the state of the atom-field, we have to solve the equation

$$i \frac{\partial}{\partial t} |\Phi_{s,eff}(t)\rangle = \mathcal{H}_{eff} |\Phi_{s,eff}(t)\rangle, \quad (29)$$

where $|\Phi_{s,eff}(t)\rangle$ denotes the state of the atom-field system after time t . The general solution may be given by

$$|\Phi_{s,eff}(t)\rangle = \sum_{n=0}^{\infty} q_n [C_g^n(t)|g, n\rangle + C_e^n(t)|e, n\rangle] \quad (30)$$

with

$$C_g^n(t) = D_1^n(t)C_g(0) + D_2^n(t)C_e(0), \quad (31)$$

$$C_e^n(t) = D_2^n(t)C_g(0) + D_3^n(t)C_e(0). \quad (32)$$

Here we have set

$$D_1^n(t) = \frac{g_2^2}{G} + \frac{g_1^2}{G} \exp\left(\frac{inf^2(n)G}{\Delta} t\right), \quad (33)$$

$$D_2^n(t) = -\frac{g_1g_2}{G} + \frac{g_1g_2}{G} \exp\left(\frac{inf^2(n)G}{\Delta} t\right), \quad (34)$$

$$D_3^n(t) = \frac{g_1^2}{G} + \frac{g_2^2}{G} \exp\left(\frac{inf^2(n)G}{\Delta} t\right). \quad (35)$$

When $4\bar{n}f^2(\bar{n}) \ll \Delta^2/G$ namely Δ be enough large and $(\bar{n}f^2(\bar{n}))^2G^2t/\delta^3 \ll \pi$ which means that evolving time is not too long, one has:

$$\frac{\Lambda_n}{\Delta} \rightarrow \frac{1}{2}, \quad (36)$$

$$\Lambda_n - \frac{\Delta}{2} \rightarrow \frac{nf^2(n)G}{\Delta}. \quad (37)$$

Therefore, by using the two above approximations we have:

$$A_k^n(t) \rightarrow D_k^n(t), \quad k = 1, 2, 3 \quad (38)$$

$$B_i^n(t) \rightarrow 0, \quad i = 1, 2, \quad (39)$$

and consequently,

$$|\Phi_{s,I}(t)\rangle \rightarrow |\Phi_{s,eff}(t)\rangle. \quad (40)$$

As it is observed, we established that in the nonlinear degenerate Raman interaction, whenever Δ be enough large, the state vectors given in (18) and (30) will be equal. Therefore, in the nonlinear degenerate Raman interaction, when Δ be enough large, the nonlinear modified effective Hamiltonian is equivalent to the nonlinear Hamiltonian in the interaction picture.

4 Generating of GKCSs via the intensity-dependent degenerate Raman interaction

Now, in the remainder of the paper, we want to present our theoretical scheme for generating temporal stable GKCSs via the introduced intensity-dependent degenerate Raman interaction, while the atom-field interaction is described by the modified effective Hamiltonian given in (13). It is worth to notice that, without loss of generality, $g_1 = g_2 = g$ is assumed in (13) for continuing the calculational works, thus one has $\lambda_1 = \lambda_2 = \lambda = g^2/\Delta$. So, one arrives at the modified effective Hamiltonian as follows:

$$\mathcal{H}_{eff} = -\lambda A^\dagger A (|g\rangle\langle e| + |e\rangle\langle g| + |g\rangle\langle g| + |e\rangle\langle e|). \quad (41)$$

Using the expression of A, A^\dagger , it may be readily seen that the effective Hamiltonian in (41) can be explicitly re-written as:

$$\mathcal{H}_{eff} = -\lambda f^2(n) a^\dagger a (|g\rangle\langle e| + |e\rangle\langle g| + |g\rangle\langle g| + |e\rangle\langle e|). \quad (42)$$

Inserting $\lambda_1 = \lambda_2 = \lambda$ in (11) and then comparing the recasted relation with (42), the above development can be, in a sense, considered simply as generalizing λ to $\lambda f^2(n)$. This result may be viewed as intensity-dependent atom-field coupling.

Now, suppose that the cavity field is initially prepared in the nonlinear CS introduced in (2). Also, the flux of the atoms injected into the cavity is such a low that there exists at most one atom at a time inside the cavity. If the first atom be initially in a superposition of $|e\rangle$ and $|g\rangle$, such that

$$|\Psi_a^{(1)}(0)\rangle = \frac{1}{\sqrt{1 + |\varepsilon_1|^2}}(|e\rangle + \varepsilon_1|g\rangle), \quad (43)$$

the state of the atom-field system at $t = 0$ is thus given by:

$$|\Psi_s^{(1)}(t=0)\rangle = \frac{1}{\sqrt{1 + |\varepsilon_1|^2}} \sum_{n=0}^{\infty} C_n (|e, n\rangle + \varepsilon_1 |g, n\rangle). \quad (44)$$

Using the time evolution of the state of atom-field described by:

$$i \frac{\partial}{\partial t} |\Psi_s^{(1)}(t)\rangle = \mathcal{H}_{eff} |\Psi_s^{(1)}(t)\rangle, \quad (45)$$

with \mathcal{H}_{eff} in (42) and the initial state in (44), the state of the atom-field system after time t is given by:

$$\begin{aligned} |\Psi_s^{(1)}(t)\rangle &= \frac{1}{2\sqrt{1 + |\varepsilon_1|^2}} \sum_{n=0}^{\infty} C_n \{ [(1 + \varepsilon_1)e^{2i\lambda n f^2(n)t} - (1 - \varepsilon_1)] |g, n\rangle \\ &\quad + [(1 + \varepsilon_1)e^{2i\lambda n f^2(n)t} + (1 - \varepsilon_1)] |e, n\rangle \}. \end{aligned} \quad (46)$$

The atomic velocities can be controlled such that every atom interacts for a fixed time τ , with the radiation field inside the cavity. Suppose that the atom which is exiting the cavity is detected in the state $|e\rangle$, so the cavity field certainly collapses to

$$|\Psi_F^{(1)}(\tau)\rangle = \mathcal{N}_1 \sum_{n=0}^{\infty} C_n \left[(1 + \varepsilon_1) e^{2i\lambda n f^2(n)\tau} + (1 - \varepsilon_1) \right] |n\rangle, \quad (47)$$

where \mathcal{N}_1 is an appropriate normalization factor that may be determined. Inserting C_n from (2) in (47) one can rewrite the field state as

$$\begin{aligned} |\Psi_F^{(1)}(\tau)\rangle = \mathcal{N}_1 \{ & (1 + \varepsilon_1) \mathcal{N}(|z|^2) \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!} [f(n)]!} e^{2i\lambda n f^2(n)\tau} |n\rangle \\ & + (1 - \varepsilon_1) \mathcal{N}(|z|^2) \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!} [f(n)]!} |n\rangle \}. \end{aligned} \quad (48)$$

It is worth mentioning the fact that the normalization coefficient $\mathcal{N}(|z|^2)$ for both non-linear CS and GKCS is exactly the same. Noticing that $n f^2(n) = e_n$, at last the state in (48) can be written as

$$|\Psi_F^{(1)}(\tau)\rangle = \mathcal{N}_1 [(1 + \varepsilon_1) |z, \alpha_1\rangle + (1 - \varepsilon_1) |z, f\rangle], \quad (49)$$

where $|z, \alpha_1\rangle$ is a GKCS introduced in (5) with $\alpha_1 \equiv -2\lambda\tau$. It is clear from (49) that we have arrived at a superposition of GKCS and initial nonlinear CS. Now, if one prepares the atom in the initial state (43) with $\varepsilon_1 = 1$, namely the probabilities of the presence of the atom in $|e\rangle$ and $|g\rangle$ are equal, the cavity field will collapse to $|z, \alpha_1\rangle$. So, we have successfully generated a temporal stable state from initial nonlinear CS $|z, f\rangle$ that is not temporally stable.

Now, assume that the second atom is injected into the cavity, while it is initially in the state described by

$$|\Psi_a^{(2)}(0)\rangle = \frac{1}{\sqrt{1 + |\varepsilon_2|^2}} (|e\rangle + \varepsilon_2 |g\rangle). \quad (50)$$

If the second atom also interacts for a time τ , with the cavity field which is described by (47) and then, it is detected in the state $|e\rangle$, the cavity field collapses to

$$\begin{aligned} |\Psi_F^{(2)}(\tau)\rangle = \mathcal{N}_2 \sum_{n=0}^{\infty} C_n \{ & (1 + \varepsilon_1)(1 + \varepsilon_2) e^{4i\lambda n f^2(n)\tau} \\ & + 2(1 - \varepsilon_1 \varepsilon_2) e^{2i\lambda n f^2(n)\tau} + (1 - \varepsilon_1)(1 - \varepsilon_2) \} |n\rangle, \end{aligned} \quad (51)$$

where again \mathcal{N}_2 is a normalization constant. Similar to the procedures we followed in (47)-(49), the state in (51) can be re-written as

$$\begin{aligned} |\Psi_F^{(2)}(\tau)\rangle = \mathcal{N}_2 [& (1 + \varepsilon_1)(1 + \varepsilon_2) |z, \alpha_2\rangle + 2(1 - \varepsilon_1 \varepsilon_2) |z, \alpha_1\rangle \\ & + (1 - \varepsilon_1)(1 - \varepsilon_2) |z, f\rangle], \end{aligned} \quad (52)$$

where we have set $\alpha_2 \equiv -4\lambda\tau$. Now, once again when the first and second atoms are initially prepared in $\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)$, i.e., $\varepsilon_1, \varepsilon_2 = 1$, the cavity field would be in $|z, \alpha_2\rangle$, i.e., in the GKCS.

By further iteration of the above procedure it is easy to show that if the m th atom is initially in the state

$$|\Psi_a^{(m)}(0)\rangle = \frac{1}{\sqrt{1 + |\varepsilon_m|^2}}(|e\rangle + \varepsilon_m|g\rangle), \quad (53)$$

and we enter N atoms into the cavity, one by one, we can generate a superposition of N temporally stable GKCSs and the initial state of the cavity field $|z, f\rangle$ that is not temporally stable. When all of N atoms that are exiting one by one from the cavity are detected in the state $|e\rangle$, the state of the cavity field is given by

$$|\Psi_F^{(N)}(\tau)\rangle = \mathcal{N}_N \left[\sum_{m=1}^N C_m(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N) |z, \alpha_m\rangle + C_0(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N) |z, f\rangle \right], \quad (54)$$

where \mathcal{N}_N is an appropriate normalization constant and $\alpha_m \equiv -2^m\lambda\tau$. Again, if $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N = 1$, then $C_0 = 0$ and $\{C_m\}_{m=1}^{N-1} = 0$. As a result, the cavity field would be in $|z, \alpha_N\rangle$ which is exactly in the GKCSs family.

We end this section with noticing that, the initial state of the cavity, i.e., nonlinear CS may be generated experimentally. As an evidence recall that the generation of photon-added coherent states as a well-known class of nonlinear CSs reported by Zavatta et al in [30]. Also, a theoretical scheme for generation of any class of nonlinear CSs in a micromaser using the intensity-dependent Jaynes-Cummings model was introduced in [31].

5 Summary and concluding remarks

In summary, using the intensity-dependent degenerate Raman interaction procedure has been introduced in the present paper and assuming the cavity field to be initially in the nonlinear CS (which is not temporally stable), the atoms that are in a superposition of the states $|g\rangle$ and $|e\rangle$ are sent inside the cavity, one by one. The atomic velocity can be controlled such that every atom interacts with the cavity field for a specific time interval τ . If each of the atoms, which is exiting from the cavity is detected in the state $|e\rangle$, the cavity field collapses to a superposition of GKCSs and initial nonlinear CS. If N atoms are sent inside the cavity, superposition of N GKCSs and the initial state of the cavity will be generated. Preparing the initial state of the atom, such that the probabilities of being every atom in states $|g\rangle$ and $|e\rangle$ are equal, then the generated states reduce to a GKCS. In fact, we have generated temporally stable GKCS from a nonlinear CS which does not have this property. In addition to these, we have found a more deep insight to

the physical foundation of GKCSs, where we observed that according to our proposal, the parameter α in the phase factor of GKCSs in (5) explicitly depends on λ (the effective coupling constant of the atom-field) and τ (the time of the interaction between atom and field). Altogether, our proposal may be considered as the first step in producing the GKCSs. We hope that the proposed theoretical scheme will stimulate further better approaches to study the GKCSs, experimentally.

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FIGURE CAPTIONS

FIG. 1 Schematic diagram of degenerate Λ -type three-level atom interaction with the single-mode cavity field.